Journal of Machine Engineering, 2024, Vol. 24, No. 1, 29–36
ISSN 1895-7595 (Print) ISSN 2391-8071 (Online)

Received*: 30 January 2024* / Accepted: *10 March 2024* / Published online: *13 March 2024*

vibration, supporting fixture, stiffness

Raiki NUMATA[1*],
Kotaro MORI[1],
Atsushi MATSUBARA[1]

QUANTITATIVE EVALUATION ON DYNAMICS OF FIXTURING SYSTEM

When complex and thin-walled workpieces produced for the aircraft industry are machined, they are supported by fixtures to prevent them from easily deforming or vibrating. To support the workpieces effectively, the stiffness of the fixture should be sufficiently high. However, the criteria required for the fixture dynamics to effectively support a workpiece during machining have not been thoroughly investigated. To minimize trial and error, the design parameters required for the fixture should be determined theoretically. Accordingly, this study proposes a method for theoretically determining the design parameters of a fixturing system. The effect of the substructure thickness on the dynamics of the entire structure was evaluated quantitatively using a theoretical model, and the validity of the model was verified experimentally. The stiffness of the entire fixturing system was estimated using the reacceptance coupling method. In addition, the relationship between the thickness of the substructure and stiffness of the entire structure was evaluated.

1. INTRODUCTION

In the aircraft industry, the machining of complex and thin-walled workpieces is necessary [1]. Because thin-walled workpieces easily deform or vibrate as they are machined, several techniques have been proposed to machine them more effectively. For example, a method for controlling the machining conditions and measuring the tool condition in real time was proposed to avoid chattering [2]. However, this reduces the cutting efficiency because the feed rate and rotational speed are adjusted. To perform machining without changing the machining conditions, the milling stability must be improved. Accordingly, workpieces are often supported by fixtures to improve the milling stability.

A sufficiently stiff supporting fixture is crucial for effectively stabilizing the workpiece. To increase the stiffness of the fixturing system, previous studies have proposed using a magnetic field [3] and piezoelectric-mechanical actuation [4]. However, these methods require specialized equipment. Although techniques that use special devices provide superior support to workpieces, conventional supporting fixtures that support workpieces via point,

[1] Micro-Engineering, Kyoto University, Japan
[*] E-mail: numata.raiki.36s@st.kyoto-u.ac.jp
https://doi.org/10.36897/jme/185894

line, or surface contacts are still used. To support workpieces effectively with conventional supporting fixtures, the placement of the supporting fixture is important. These conventional supporting fixtures consist of a component in contact with the workpiece and other components that move the supporting fixture to the desired position. Therefore, the stiffness of the substructure that shift the supporting fixture into the desired position is important. However, the criteria required for the substructure dynamics to effectively support a workpiece while it is machined have not been thoroughly investigated. To minimize trial and error, the dynamic characteristics required for the substructure should be determined theoretically.

In this study, a method for determining the appropriate design parameter of a fixturing system is proposed. The method facilitates the determination of design parameter. Effect of the stiffness of the design substructure on the stiffness of the entire fixturing system is evaluated quantitatively.

The rest of this paper is structured as follows. Section 2 discusses the concept of the supporting system, Section 3 describes the experiments conducted to investigate the stiffness of the fixturing system, Section 4 explains the method used to estimate the stiffness of the fixture, Section 5 presents the relationship between the thickness of the backplate and the stiffness of the entire fixture, and Section 6 contains the conclusion of the study.

2. CONCEPT

A supporting system consists of two parts: (1) a contact element, which is designed to support the workpiece with the desired stiffness, and (2) a backbone element that shifts the contact element to the desired position. The dynamics of the backbone element should be properly designed to prevent the deterioration of the dynamics of the contact elements.

This study investigated the support system shown in Fig. **1**. The head and backplate corresponded to the contact and backbone elements, respectively. In this system, the backplate structure was sufficiently simple to be considered a cantilever beam. The backplate thickness must be appropriately designed to maximize the stiffness of the supporting system.

A theoretical model of the fixturing system describing the effect of the thickness of the substructure on the stiffness of the entire fixture is now presented. The stiffness of the entire fixture k_{rs} can be expressed by coupling the backplate stiffness and head stiffness according to

$$k_{\mathrm{rs}} = \frac{k_{\mathrm{bp}} k_{\mathrm{hd}}}{k_{\mathrm{bp}} + k_{\mathrm{hd}}}, \tag{1}$$

where k_{bp} is the stiffness of the backplate and k_{hd} is the stiffness of the head. The value of k_{bp} can be calculated using the cantilever beam deflection formula, which is expressed as

$$k_{\mathrm{bp}} = \frac{3EI}{(l-a)^3}, \tag{2}$$

$$I = \frac{bh^3}{12}, \tag{3}$$

where E is Young's modulus, I is the moment of inertia of the area, l is the length of the backplate, a is the distance from the free end of the cantilever beam to the point at which the head is attached, b is the width of the backplate, and h is the thickness of the backplate. Using Eqs. 1–3, the relationship between the stiffness of the entire fixture k_{rs} and the thickness of the backplate h can be written as

$$k_{\mathrm{rs}} = \frac{3Ebh^3 k_{\mathrm{hd}}}{3Ebh^3 + 12(l-a)^3 k_{\mathrm{hd}}}. \tag{4}$$

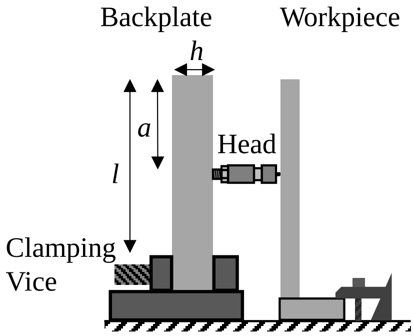

Fig. 1. Schematics of the entire fixture supporting workpiece

The stiffness of the entire fixture k_{rs} was experimentally determined by decoupling the dynamics of the workpiece from the entire supporting system via the reacceptance coupling (RC) method. The latter is used to estimate the dynamics of an entire vibration system by coupling the dynamics of its substructures as derived from measurements or numerical calculations [5]. When vibration characteristics of the entire fixture and the workpiece are linear, the RC method can be expanded to estimate frequency response of the entire fixture [6]. In the RC method, the dynamics that the entire fixture exhibits as it supports the work-piece is estimated. The dynamics of the entire vibration system shown in Fig. **2** can be estimated using the RC method via

$$G_{\mathrm{ij}} = R_{\mathrm{ij}} - R_{\mathrm{ia}}(R_{\mathrm{aa}} + RS_{\mathrm{a}})^{-1} R_{\mathrm{aj}}, \tag{5}$$

where G_{ij} is the compliance of the entire vibration system from the excitation force at #j to the displacement at #i, R_{ij} is the compliance of the workpiece without a fixture from the excitation force at #j to the displacement at #i, and RS_{a} is the compliance of the fixture attached at #a. When R_{ij} and G_{ij} are known, RS_{a} can be decoupled according to

$$RS_a = R_{a2}(R_{22} - G_{22})^{-1} R_{2a} - R_{aa}. \tag{6}$$

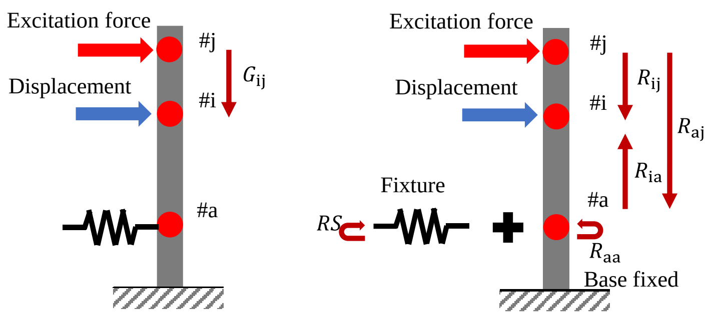

Fig. 2. Excitation response of the entire vibration system, fixture, and workpiece

3. EXPERIMENTS INVESTIGATING THE STIFFNESS OF THE ENTIRE FIXTURE

Hammer tests were conducted to obtain the compliances used to estimate the stiffness of the entire fixture, as described in Section 4. In addition, the compliances of the backplates were obtained to assess their stiffness values for different thicknesses. Fig. **3** shows schematics of the vibration system used in this study, which consisted of the workpiece and supporting fixture. The supporting fixture was placed at point 2 on the workpiece. A structural carbon steel plate (JIS S50C, thickness: 10 mm, width: 100 mm, and height: 200 mm) was used as the workpiece. To minimize the effect of the clamping device, the bottom of the workpiece was fixed using two bolts. The supporting fixture consisted of a backplate and head. The backplate was made of a structural carbon steel plate. Backplates of different sizes were used; each had a width of 20 mm and height of 200 mm, and different thicknesses of 10, 13, 15, 18, 20, and 30 mm. The head consisted of a spherical positioning pin made of poly-acetal, two cylindrical parts (JIS S45CD), and a force sensor (kistler9001A). The head was placed in a position normal to the mounting holes on the backplate using bolts and nuts. The pressing force exerted by the head on the workpiece was adjusted to 50 N using a force sensor on the head by changing the stick out of the bolt.

Excitation was conducted using an impact hammer, and accelerations were measured using a PCB 313B15. Then the compliances were calculated using an FFT analyzer (ONO Sokki DS-5000). The sampling frequency was 5000 Hz with 4096 sampling points, and the measurement was repeated five times for averaging. All hammer tests were performed using the same settings. The following compliance was measured during the experiment:

1) Compliance of the workpiece itself without a supporting fixture:
   a) From excitation force at point 1 to displacement at point 1 (R_{11}),
   b) From excitation force at point 1 to displacement at point 2 (R_{21}),
   c) From excitation force at point 2 to displacement at point 1 (R_{12}),
   d) From excitation force at point 2 to displacement at point 2 (R_{22}).
2) Compliance of the entire vibration system with different backplates:
   a) From excitation force at point 1 to displacement at point 1 (R_{11})
3) Compliance of the backplates with different thicknesses:
   a) From head attachment point to the same point.

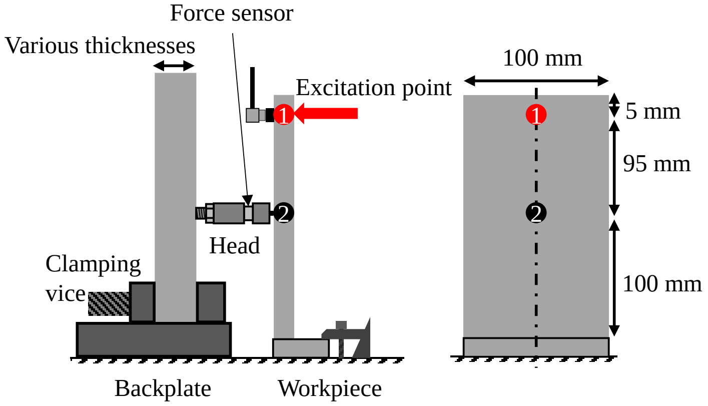

Fig. 3. Schematics of the entire vibration system and workpiece

Figure 4 shows the measured compliance of the workpiece by itself and that of the workpiece supported by a fixture with a 30 mm thick backplate from the excitation force at point 1 to the displacement at point 1. The figure indicates that the supporting fixture had a significant effect on the dynamics of the workpiece. Fig. **5** shows the measured compliances of six different backplates. The magnitude of the backplate compliance at the first natural frequency tended to decrease as the backplate thickness increased. These results indicate that the backplate stiffness increased when the backplate thickness increased.

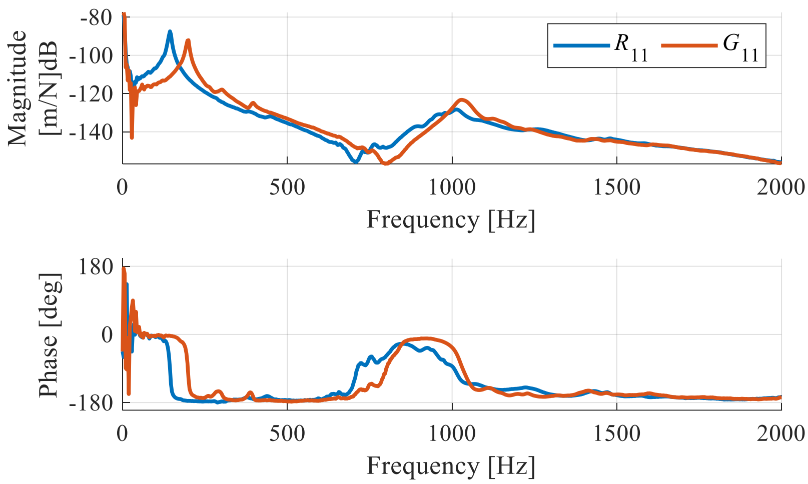

Fig. 4. Compliance of workpiece by itself (R_{11}) and workpiece supported by a fixture with a 30 mm thick backplate (G_{11})

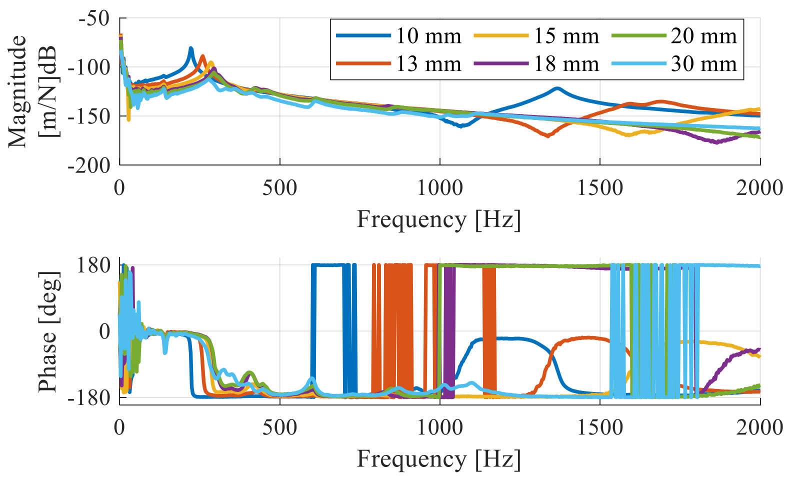

Fig. 5. Compliances of backplates with different thicknesses

4. ESTIMATION OF THE STIFFNESS OF THE ENTIRE FIXTURE

In this section, the compliance of the entire fixture RS_{a} is estimated via the RC method using the measured compliance of the workpiece R_{ij} and the measured compliance of the entire vibration system G_{ij}. Using Eq. 6, the value of RS_a was estimated for the backplates with thicknesses of 10, 13, 15, 18, 20, and 30 mm. Fig. **6** shows the estimated compliance of the entire fixture for each of the six backplate thicknesses.

From 80 to 200 Hz and 400 to 900 Hz, the magnitude of the compliance of the entire fixture was approximately constant, and the phase was approximately 0°. These results show that, within these frequency ranges, the entire fixture had the same vibrational characteristics as a spring. However, from 200 to 400 Hz, the compliance of the entire fixture did not remain constant; it decreased as the backplate thickness increased. Therefore, the change in the compliance of the entire fixture from 200 to 400 Hz can be minimized by increasing the thickness of the backplate.

The static stiffness of the entire fixture was calculated using the compliance at 0 Hz. However, the estimated compliance at approximately 0 Hz is unreliable because of the large uncertainty of the accelerometer measurements at frequencies close to 0 Hz. Therefore, the compliance below 200 Hz was assumed to be constant, and the stiffness of the entire fixture was calculated using the average compliance between 93.75 Hz and 165.625 Hz. The stiffness values calculated for the entire fixture are listed in Table 1, which indicates that the stiffness of the entire fixture increased as the backplate thickness increased. By increasing the backplate thickness from 10 mm to 15 mm, the stiffness of the entire fixture increased by 3.6×10^5 N/m. However, the stiffness of the entire fixture only increased by 0.8×10^5 N/m as the thickness of the backplate increased from 15 mm to 20 mm. Therefore, the differential increase in the stiffness of the entire fixture decreased as the thickness of the backplate increased.

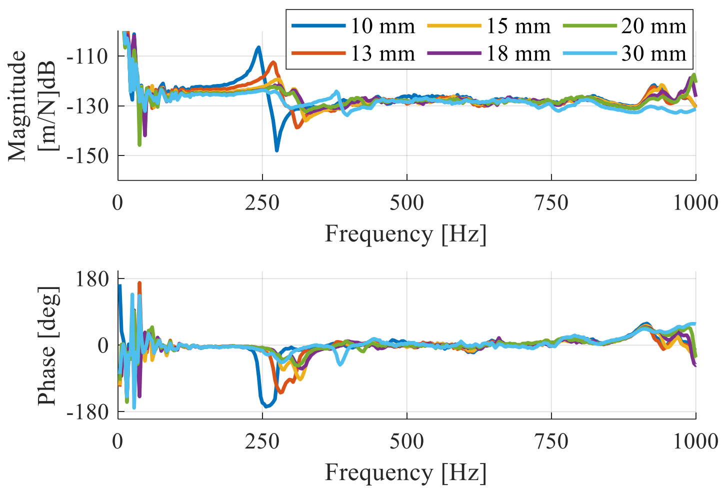

Fig. 6. Estimated compliance of the entire fixture for different backplate thicknesses

Table 1. Stiffnesses of the entire fixture calculated by averaging the compliance between 93.75 Hz and 165.625 Hz

Thickness of Backplate (mm)	Calculated stiffness of entire fixture (N/m)
10	1.41×10^6
13	1.56×10^6
15	1.77×10^6
18	1.85×10^6
20	1.85×10^6
30	1.94×10^6

5. RELATIONSHIP BETWEEN THICKNESS OF THE BACKPLATE AND STIFFNESS OF THE ENTIRE FIXTURE

In this section, the relationship between the compliance of the entire fixture (estimated using the RC method) and the thickness of the backplate is evaluated. Fig. **7** shows the relationship between the backplate thickness and the stiffness of the entire fixture; the blue dots represent the experimental data, and the red line represents the theoretical stiffness of the entire fixture calculated using Eq. 4. The parameters of the theoretical model are summarized in Table 2. The stiffness of the head was assumed to be equal to the stiffness of the entire fixture with a 30-mm thick backplate.

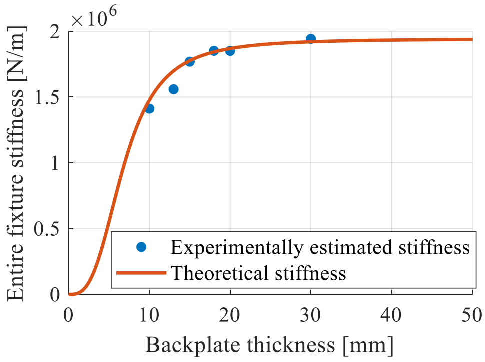

Fig. 7. Experimental (blue dots) and theoretical (red line) compliance of the entire fixture as a function of the backplate thickness

Table 2. Model parameters for the theoretical relationship between the backplate thickness and the stiffness of the entire fixture

Young's modulus *E*	205 GPa
Length of backplate *l*	175 mm
Width of backplate *b*	20 mm
Distance from free end to head attachment point *a*	120 mm
Thickness of backplate *h*	0-50 mm
Stiffness of head k_{hd}	1.94×10^6 N/m

The experimentally estimated stiffness agreed with the theoretical model. Although the stiffness of the entire fixture tended to increase as the backplate thickness increased, the differential change decreased. The theoretical values indicated that the overall stiffness of the entire fixture improved only slightly when the backplate thickness exceeded 30 mm. For the setup used in this experiment, the stiffness of the entire fixture with a backplate thickness of 18 mm was 95% that of the entire fixture with a backplate thickness of 30 mm. Based on these results, a backplate with a thickness of 18 mm was sufficient for this experimental setup. Therefore, the proposed method provides the design parameters required for determining the necessary backplate thickness in a fixturing system used to machine complex and thin-walled workpieces in the aircraft industry.

6. CONCLUSION

In this study, a method was proposed to determine the design parameters of a fixturing system used to machine complex and thin-walled workpieces in the aircraft industry. The fixturing system was represented by a simple model, and the desired thickness of the substructure was determined using the relationship between the stiffness of the entire fixture and the thickness of the substructure. The theory assuming that the stiffness of the entire fixture is coupled to the stiffness of the substructure agreed with the experimental results. The theoretical evaluation suggested that an increase in the stiffness of the substructure had a small effect on the stiffness of the entire fixture over a certain stiffness range. For the experimental setup used in this study, a substructure with a thickness of 18 mm increased the stiffness of the entire fixture to 95% of that for a substructure with a thickness of 30 mm.

When designing a substructure, the theory proposed in this study can save time and reduce the cost of materials by its ability to identify the required thickness of the substructure. In the proposed method, the relationship between the thickness of the backplate and the stiffness of the entire structure was evaluated. Therefore, even if the geometry of the head differs, the model presented in this study can be used when the stiffness of the head remains constant. However, the proposed method includes limitation since the backplate was simplified as a cantilever beam. Also, the currently proposed method cannot be applied to backplate with multiple structures. Supporting fixture with more complex structure will be investigated in the next step.

REFERENCES

[1] M'SAOUBi R., AXINTE D., SOO S.L., NOBEL C., ATTIA H., KAPPMEYER G., ENGIN S., SIM W.M., 2015, *High Performance Cutting of Advanced Aerospace Alloys and Composite Materials*, CIRP Annals, 64/2. 557–580, https://doi.org/10.1016/j.cirp.2015.05.002.

[2] BLEICHER F., SCHORGHOFER P., HABERSOHN C., 2018, *In-Process Control with a Sensory Tool Holder to Avoid Chatter*, Journal of Machine Engineering, 18/3, 16–27, https://doi.org/10.5604/01.3001.0012.4604.

[3] WEICHENG G., YONG Z., XIAOHUI J., NING Y., KUN W., XIAO L., 2021, *Improvement of Stiffness During Milling Thin-Walled Workpiece Based on Mechanical/Magnetorheological Composite Clamping*, Journal of Manufacturing Processes, 68/A, 1047–1059, https://doi.org/10.1016/j.jmapro.2021.06.039.

[4] BAKKER O.J., POPOV A., RATCHEV S., 2008, *Control of a Workpiece Holder with Piezo-Electro-Mechanical Actuation*, Journal of Machine Engineering, 8/3, 17–28.

[5] SCHMITZ T.L., DONALSON R.R., 2000, *Predicting High-Speed Machining Dynamics by Substructure Analysis*, CIRP Annals, 49/1, 303–308, https://doi.org/10.1016/S0007-8506(07)62951-5.

[6] MORI K., MATSUBARA A., 2022, *Estimation of Supporting Fixture Receptance for Thin-Walled Milling*, CIRP Annuals, 71/1, 333–336, https://doi.org/10.1016/j.cirp.2022.04.038.